Name: Solutions

Work in groups to answer as many problems as you can. Ask questions if you get stuck. The numbers used on this worksheet may require a calculator. Keep in mind that numbers you will have on exams will be nice enough to do without a calculator.

1. A jeweler has five rings, each weighing 18 grams, made of an alloy of 10% silver and 90% gold. He decides to melt down the rings and add enough silver to reduce the gold content to 75%.

   (a) Construct a model that gives the fraction of the new alloy that is pure gold. (Let x represent the number of grams of silver added)

   $$G(x) = \frac{\text{Amount of gold}}{\text{Total weight}} = \frac{0.9 \times 5 \times 18}{5 \times 18 + x}$$

   $$= \frac{\text{Amount of gold}}{\text{Initial weight} + \text{Silver added}}$$

   Answer: $G(x) = \dfrac{81}{90 + x}$

   (b) How much pure silver must be added for the mixture to have a gold content of 75%?

   $$\frac{81}{90+x} = \frac{3}{4} \Longrightarrow 324 = 270 + 3x \Longrightarrow 54 = 3x$$

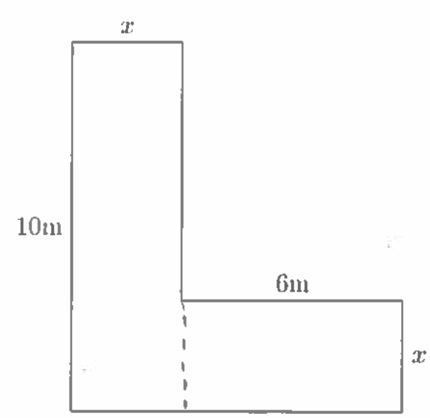
   Answer: $x = 18$.

2. An architect is designing a building whose footprint has the shape shown.

   (a) Construct a model that gives the total area of the footprint of the building.

   $10x + 6x$

   Answer: $A(x) = 16x$



(b) Find x such that the area of the building is 144m^2.

$$16x = 144$$
$$x = \frac{144}{16} = \frac{3\cdot4\cdot3\cdot4}{4^2}$$

Answer: $x = 9$

3. Find the point (i.e. x *and* y value) where the two lines intersect.

(a) $y_1 = 4 - x$; $y_2 = x$

$$4 - x = x$$
$$4 = 2x$$

Answer: $(2,2)$

(d) $y_1 = x - 3$; $y_2 = 7 - x$

$$x - 3 = 7 - x$$
$$2x = 10$$

Answer: $(5,2)$

(b) $y_1 = 3 - \frac{4}{3}x$; $y_2 = \frac{2}{3}x - 3$

$$3 - 4/3x = 2/3x - 3$$
$$6 = 2x$$

Answer: $(3,-1)$

(e) $y_1 = -\frac{3}{2}x$; $y_2 = -4 - \frac{1}{2}x$

$$-\frac{3}{2}x = -4 - \frac{1}{2}x$$
$$4 = x$$

Answer: $(4,-6)$

(c) $y_1 = \frac{5}{3} - \frac{1}{3}x$; $y_2 = 2x - 3$

$$\frac{5}{3} - \frac{1}{3}x = 2x - 3$$
$$\frac{14}{3} = \frac{7}{3}x$$

Answer: $(2,1)$

(f) $y_1 = 7 - x$; $y_2 = \frac{2}{3}x + \frac{1}{2}$

$$7 - x = \frac{2}{3}x + \frac{1}{2}$$
$$\frac{13}{2} = \frac{5}{3}x$$

Answer: $\left(\frac{39}{10}, \frac{31}{10}\right)$

4. Dietmar is in the process of choosing a cell phone and a cell phone plan. The first plan charges \$0.20 per minute plus a monthly fee of \$10, and the second plan offers unlimited minutes for a monthly fee of \$100.

(a) Find a linear function that models the models the monthly cost $f(x)$ of the first plan in terms of the number x of minutes used.

Answer: $f(x) = 10 + 0.2x$

(b) Find a linear function that models the models the monthly cost $g(x)$ of the second plan in terms of the number x of minutes used.

Answer: $g(x) = 100$

(c) Determine the number of minutes for which the two plans have the same monthly cost.

$100 = 10 + 0.2x$

Answer: $x = 450$

5. Lina is considering installing solar panels on her house. Solar Advantage offers to install solar panels that generate 320kWh of electricity per month for an installation fee of \$15,000. She uses 350kWh of electricity per month, and her local utility company charges \$0.20 per kWh.

   (a) If Lina gets all her electrical power from the local utility company, find a linear function U that models the cost $U(x)$ of electricity for x months service.

   Cost = cost per Watt × # of whats

   Answer: $U(x) = 70x$

   (b) If Lina has Solar Advantage install solar panels on her roof that generate 320kWh of power per month, find a linear function S that models the cost $S(x)$ of electricity for x months service.

   Cost = Initial + cost per Watt × (# of watts − 320)

   Answer: $S(x) = 15{,}000 + 6x$

   (c) Determine the number of months it would take to reach the break-even point for the installation of Solar Advantages solar panels.

$$70x = 15{,}000 + 6x$$
$$64x = 15{,}000$$
$$x = \frac{1875}{8} = \frac{1600+240+32+3}{8} = 200+30+4+\frac{3}{8}$$

Answer: 234.375 months

Cont.

6. Kofi wants to buy a new car, and he has narrowed his choices to two models.

Model A sells for \$15,500, gets 25mi/gal, and costs \$350 a year for insurance
Model B sells for \$19,100, gets 48mi/gal, and costs \$425 a year for insurance

Kofi drives about 36,000 miles per year, and gas costs about \$4.50 a gallon.

(a) Find a linear function A that models the total cost $A(x)$ of owning Model A for x years.

$$A(x) = \text{initial} + \left(\text{insurance} + \frac{\text{\# of miles}}{\text{miles per gallon}} \times \text{cost per gallon}\right)x$$

$$= 15500 + \left(350 + \frac{36000}{25} \cdot (4.5)\right)x$$

$$= 15500 + \left(350 + \frac{36000}{25} \cdot \frac{9}{2}\right)x$$

Answer: $A(x) = 15500 + 6830x$

(b) Find a linear function B that models the total cost $B(x)$ of owning Model B for x years.

$$B(x) = 19100 + \left(425 + \frac{36000}{48} \cdot \frac{9}{2}\right)x$$

Answer: $B(x) = 19100 + 3800x$

(c) Find the number of years of ownership for which the cost to Kofi of owning Model A equals the cost of owning Model B.

$$19100 + 3800x = 15500 + 6830x$$

$$3600 = 3030x$$

Answer: 1.19 years.

Cont.

7. The bacteria *Streptococcus pneumoniae* is the cause of many human diseases, the most common being pneumonia. A culture of these bacteria initially has 10 bacteria and increases at a rate of 26% per hour.

(a) Find the hourly growth factor a, and find an exponential model $f(t) = Ca^t$ for the bacteria count in the sample, where t is measured in hours.

$a = 1.26$

Answer: $f(t) = 10(1.26)^t$

(b) Use the model you found to predict the number of bacteria in the sample after 5 hours.

$10(1.26)^5$

Answer: 31.76

(c) Find a model that represents the number of bacteria in the model after t *minutes*.

Answer: $10(1.26)^{t/60}$

(d) Find a model that represents the number of bacteria in the model after t *days*.

Answer: $10(1.26)^{24t}$

8. Kerri invested \$2000 on January 1, 2005, in a mutual fund that for the past 5 years had a yearly increase of 5%. Assume that the mutual fund continues to grow at 5% each year.

(a) Find an exponential growth model for Kerri's investment t years since 2005. What is the growth factor?

$a = 1.05$

Answer: $I(t) = 2000(1.05)^t$

(b) Use the model found to predict Kerri's investment in 2010.

Answer: \$2552.56

Challenge (c) Kerri finds that a different fund is offering an increase of 10% per year if you invest \$1000. If Kerri opens this in 2010 and uses \$1000 of her investment in the current fund to open it, in what year will the new fund have more money than the first fund?

$1552.56(1.05)^t = 1000(1.1)^t$

$1.55256 = \left(\frac{1.1}{1.05}\right)^t$ ← Not learned these yet.

Answer:

Cont.

9. In 2006 the U.S. "housing bubble" was beginning to burst. One news article at the time stated: "The median price of a home sold in the United States in the third quarter of 2006 was \$232,000 and forecasters predict that the median will fall by 8.9% each year."

(a) Find the decay factor a, and use it to find an exponential growth model $E(x) = Ca^x$ for the median price of a home sold in the United States x years since the prediction.

$a = 0.911$

Answer: $E(x) = 232000\,(0.911)^x$

(b) Use the model in part (a) to find the median price of homes sold in the third quarter of 2009.

$x = 3$

Answer: 175405.46

(c) Find a model that represents the median price of a home sold x *months* after the prediction.

Answer: $E(x) = 232000\,(0.911)^{x/12}$

(d) Find a model that represents the median price of a home sold x *quarters* after the prediction.

Answer: $E(x) = 232000\,(0.911)^{x/4}$

10. The Centers for Medicare and Medicaid Services report that health-care expenditures per capita were \$2813 in 1990 and \$3329 in 1993. Assume that this rate of growth continues.

(a) Find the three year growth factor a and use it to find an exponential growth model $E(x) = Ca^x$ for the annual health-care expenditures per capita, where x is the number of three-year time periods since 1990.

$a = \frac{3329}{2813} = 1.18$

Answer: $E(x) = 2813\,(1.18)^x$

(b) Use this model to predict the expenditures per capita in 1996 and 2005

$x = 2$
$x = 5$

Answer: \$3916.82

Answer: \$6435.46

(c) Find a model that represents the expenditure per capita, where x is the number of *years* since 1990.

Answer: $2813(1.18)^{x/3}$

Cont.

11. Although India occupies only a small portion of the world's land area, it is the second most populous country in the world, and its population is growing rapidly. The population was 846 million in 1990 and 1148 million in 2000. Assume that India's population grows exponentially.

(a) Find the 10-year growth factor and the annual growth factor for India's population.

$a^{10} = \frac{1148}{846} = 1.36$

Answer: $a^{10} = 1.36$

Answer: $a = (1.36)^{1/10}$

(b) Find an exponential model P for the population t years after 1990.

Answer: $P = 846(1.36)^{t/10}$

(c) Use the model to predict the population in 2010.

$t = 20$

Answer: 1564.76

12. Nuclear power plants produce radioactive plutonium-239, which has a half-life of 24,360 years. A 700 gram sample of plutonium-239 is placed in an underground waste disposal facility.

(a) Find a function that models the mass $m(t)$ of plutonium-239 remaining in the sample after t *years*. What is the decay factor?

Answer: $m(t) = 700(1/2)^{t/24360}$

Answer: $a = 1/2$

(b) Use the model to predict the amount of plutonium-239 remaining after 500 years.

$t = 500$

Answer: 690.11

Challenge: How many years will it take for there to be 100 grams of the sample left. (Think about it in half lives and go from there)

Not learned.

Answer:

13. If \$500 is invested at an interest rate of 3% per year, compounded quarterly find the value of the investment after the given number of years.

(a) 1 year

$500\left(1+\frac{0.03}{4}\right)^{4\times 1}$

Answer: 515.17

(b) 2 years

$500\left(1+\frac{0.03}{4}\right)^{4\times 2}$

Answer: 530.80

(c) 5 years

$500\left(1+\frac{0.03}{4}\right)^{4\times 5}$

Answer: 580.59

14. If \$2500 is invested at an interest rate of 2.5% per year, compounded daily find the value of the investment after the given number of years.

(a) 2 years

$2500\left(1+\frac{0.025}{365}\right)^{365\times 2}$

Answer: 2628.17

(b) 3 years

$2500\left(1+\frac{0.025}{365}\right)^{365\times 3}$

Answer: 2694.70

(c) 6 years

$2500\left(1+\frac{0.025}{365}\right)^{365\times 6}$

Answer: 2904.57

15. If \$4000 is invested at an interest rate of 1.6% per year, compounded quarterly find the value of the investment after the given number of years.

(a) 4 years

$4000\left(1+\frac{0.016}{4}\right)^{4\times 4}$

Answer: 4263.83

(b) 6 years

$4000\left(1+\frac{0.016}{4}\right)^{4\times 6}$

Answer: 4402.19

(c) 8 years

$4000\left(1+\frac{0.016}{4}\right)^{4\times 8}$

Answer: 4545.05

16. If \$10000 is invested at an interest rate of 10% per year, compounded semiannually find the value of the investment after the given number of years.

(a) 5 years

$10000\left(1+\frac{0.1}{2}\right)^{2\times 5}$

Answer: 16288.95

(b) 10 years

$10000\left(1+\frac{0.1}{2}\right)^{2\times 10}$

Answer: 26532.98

(c) 15 years

$10000\left(1+\frac{0.1}{2}\right)^{2\times 15}$

Answer: 43219.42

17. If \$3000 is invested at an interest rate of 4% per year, Find the amount of the investment at the end of 5 years for the following compounding methods.

(a) Annual

$3000\left(1+\frac{0.04}{1}\right)^{1\times 5}$

Answer: 3649.96

(b) Semiannual

$3000\left(1+\frac{0.04}{2}\right)^{2\times 5}$

Answer: 3656.98

(c) Monthly

$3000\left(1+\frac{0.04}{12}\right)^{12\times 5}$

Answer: 3662.99

(d) Daily

$3000\left(1+\frac{0.04}{365}\right)^{365\times 5}$

Answer: 3664.17

18. Find the annual percentage yield that earns;

(a) 2.5% each year, compounded daily

Answer: $\frac{0.025}{365}$

(b) 4% each year, compounded monthly

Answer: $\frac{0.04}{12}$

(c) 3.25% each year, compounded quarterly

Answer: $\frac{0.0325}{4}$

(d) 5.75% each year, compounded semiannually

Answer: $\frac{0.0575}{2}$.

19. Kai wants to invest \$5000 and he is comparing two different investment options:

- 3.25% interest per year, compounded semiannually
- 3% interest per year, compounded daily

Which of the two options would provide the better investment?

$$\left(1+\frac{0.0325}{2}\right)^2 = 1.03276$$

$$\left(1+\frac{0.03}{365}\right)^{365} = 1.03045$$

Answer: 3.25%

20. Sonya wants to invest \$3000 and she is comparing three different investment options:

- 4.5% each year, compounded semiannually
- 4.25% each year, compounded quarterly
- 4% each year, compounded daily

Which of the given interest rates and compounding periods would provide the best investment?

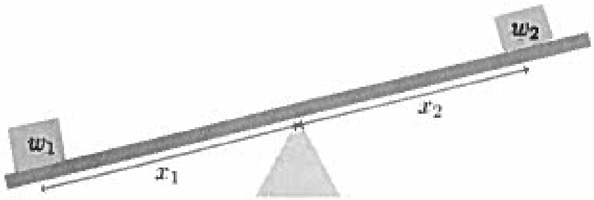
$$\left(1+\frac{0.045}{2}\right)^2 = 1.04551$$

$$\left(1+\frac{0.0425}{4}\right)^4 = 1.04318$$

$$\left(1+\frac{0.04}{365}\right)^{365} = 1.04081$$

Answer: 4.5%.

21. Challenge Problem



The figure on the left shows a lever system, similar to a seesaw you might find in a children's playground For the system to balance the product of the weight and its distance from the pivot point must be the same on each side. That is

$$w_1 x_1 = w_2 x_2.$$

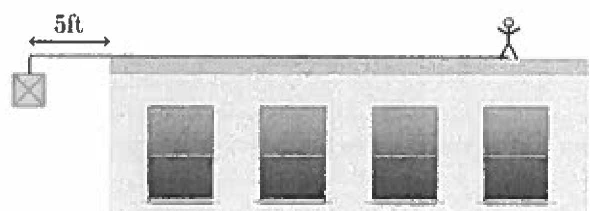
This equation is called the Law of the Lever.

(a) A mother and her son are playing on a seesaw. The boy is at one end, 8ft from the picvot point. If the boy weighs 100lbs and the mother weighs 125lbs, at what distance from the pivot point should the woman sit so that the seesaw is balanced.

$100 \cdot 8 = 125 \cdot x$

Answer: $x = 6.4$

(b) A 30ft plank rests on top of a flat roofed building, with 5ft of the plank projecting over the edge, as shown in the picture. A 240lb worker sits on one end of the plank. What is the largest weight that can be hung on the projecting end of the plank if it is to remain in balance?



$240 \cdot 25 = w \cdot 5$

$240 \cdot 5 = w$

Answer: 1200lbs.

The End.